Int. J. Heat Mass Transfer Vol. 23, pp. 329–337
Pergamon Press Ltd. 1980. Printed in Great Britain

AN IMPROVED ZONE METHOD USING MONTE CARLO TECHNIQUES FOR THE SIMULATION OF RADIATION IN INDUSTRIAL FURNACES

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(*Received* 15 *March* 1978 *and in revised form* 12 *June* 1979)

Abstract – Calculation procedures are proposed in which the Monte Carlo simulation methods can be conveniently used to compute the view factor matrix entering into Hottel's zone method. Difficulties caused by inherent statistical errors can be eliminated by regression. An empirical formula is proposed for the beam length distribution that enables the total exchange areas to be calculated for any gas absorption model (i.e. which is not restricted to an exponential absorption law) from the view factors in a diatherm medium. By this procedure the multiple integrations required to calculate the view factors are uncoupled from the temperature distributions, so that they need to be calculated only once for a given geometrical configuration.

The usefulness and applicability of the procedure is illustrated by a number of examples.

NOMENCLATURE

A, area of surface zone [m^2];
$a(i, j, k)$, coefficient in equation (2);
$\mathbf{B}$, coefficient matrix in equation (5);
$b(i, k)$, coefficient in equation (5);
$\mathbf{D}$, diagonal matrix of dispersions;
E_i, hemispherical black body emissive power of zone i [$W m^{-2}$];
$\mathscr{F}$, term in equation (19);
F_{ij}, view factor from zone i to zone j;
$\mathbf{I}, \mathbf{I}'$, unity vectors;
k_i, absorption factor of volume zone i [m^{-1}];
m, number of surface zones;
$N(y)$, group in the empirical expression for the beam length distribution model;
n, number of volume zones;
$P(y)$, polynomial in the empirical expression for the beam length distribution;
Q_i, non radiative heat flux leaving zone i [W];
S, beam length between emission and first surface contact [m];
$\overline{g_1 g_2}$, direct interchange area from volume g_1 to volume g_2 [m^2];
$\overline{sg}$, idem from surface s to volume g [m^2];
$\overline{s_1 s_2}$, idem from surface s_1 to surface s_2 [m^2];
T, absolute temperature [K];
V, volume [m^3];
X, length of the side of a square [m];
y, dimensionless beam length $= (S - S_{min})/(S_{max} - S_{min})$;
$\bar{y}$, geometric mean of y;
$\overline{Z_i Z_j}$, total interchange area from zone i to zone j [m^2];
$\overline{z_i z_j}$, direct interchange area from zone i to zone j [m^2].

Greek symbols

β, inclination with respect to the direction normal to the surface of an emitted or captured beam [rad];
ρ_{ij}, Kronecker delta ($= 1$ if $i = j$, $= 0$ if $i \neq j$);
ε_i, emissivity of surface zone i;
ρ, reflectivity;
$\tau(S)$, transmittance along a beam path S;
$\bar{\tau}_{ij}$, mean transmittance between zone j and i.

Subscripts

e, emittor;
g, gas zone;
I, imaginary surface;
r, receptor; also real gas;
R, real surface;
s, surface zone.

Superscripts

* unsmoothed value;
$\hat{}$ smoothed value.

1. INTRODUCTION

THE ZONE method for calculating temperature distributions and fluxes in furnaces was introduced by Hottel [1] and has been developed further by Hottel and Cohen [2] and Hottel and Sarofim [3, 4]. In this method the space in which the radiation has to be calculated is divided into a number of surface and volume elements or zones, each assumed uniform in

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temperature and radiation properties. The zone approach reduces the set of integro-differential equations describing the energy transfer into a set of non-linear algebraic equations. The set of energy balances for the zones in a closed radiation system in which heat transfer by conduction and convection — partly also with the surroundings — is accounted for, can be written

$$\begin{vmatrix} \overline{Z_1Z_1} - \sum_j \overline{Z_1Z_j} & \overline{Z_2Z_1} & \cdots & \overline{Z_nZ_1} \\ \overline{Z_1Z_2} & \overline{Z_2Z_2} - \sum_j \overline{Z_2Z_j} & \cdots & \overline{Z_nZ_2} \\ \vdots & \vdots & & \\ \overline{Z_1Z_n} & \overline{Z_2Z_n} & \cdots & \overline{Z_nZ_n} - \sum_j \overline{Z_nZ_j} \end{vmatrix} \begin{vmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{vmatrix} = \begin{vmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{vmatrix}. \quad (1)$$

The total exchange area $\overline{Z_iZ_j}$ is the ratio of the radiation energy emitted by zone Z_i which is absorbed by zone Z_j (directly or after reflection on other zones) and of the total hemispherical emissive power of zone Z_i [5]. According to Gebhart's terminology [6]: the total exchange area $\overline{Z_iZ_j}$ is the overall absorption factor between an emitter Z_i and a receptor Z_j, multiplied by $A_i\varepsilon_i$ for an emitting surface and $4k_iV_i$ for an emitting volume. Z_i represents a surface or a volume zone. Q_i represents the non radiative heat flux leaving Z_i.

The set of non linear algebraic equations (1) can be solved numerically for the temperatures and for the fluxes by means of Ness' rapidly converging method [7], based upon a Newton–Raphson procedure.

The sequence of calculations leading to the total exchange areas appearing in (1) may be summarized as follows. To start with, the view factors between surfaces in a transparent (non absorbing, non scattering) medium are calculated. The second step is the calculation of view factors between surfaces and volumes in a real medium and this requires accounting for the absorption, which depends upon the gas composition and the temperature distribution. The direct exchange areas between surfaces are obtained by multiplying the view factors by the emitter areas. The direct exchange areas between surfaces and volumes and among volumes can be derived from the direct exchange areas between surfaces, provided that the imaginary surfaces bounding the volumes are also accounted for. Finally, the total exchange areas are calculated by accounting for the radiation received by the receptor by both direct and reflected radiation, using the algorithms of Hottel [1].

This paper originated from efforts to computerize completely advanced methods of furnace design. A method was developed for avoiding some of the limitations caused by statistical errors arising from the applications of the Monte Carlo method to the calculation of the view factors. Further, an approach is proposed enabling the use of Monte Carlo methods in computations for real media to be uncoupled from the variations in temperatures and concentrations in the gaseous medium. This is done by means of an empirical expression for the beam length distribution. The approach enables complex gas absorption laws to be accounted for.

2. CALCULATIONS OF THE VIEW FACTORS AMONG SURFACES IN A TRANSPARENT MEDIUM

In a closed radiation system and with a transparent medium, the radiation emitted from a given surface is entirely and directly captured by the surrounding surfaces. To obtain the view factor the fraction of the total emitted radiation oriented in a given direction has to be integrated over all the emitting points and over every beam directed towards the receptor

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\beta_i \cos\beta_j}{\pi S^2} \, dA_j \, dA_i.$$

Analytical integration is possible in some particular cases only. Hottel and Cohen [2] applied graphical methods. The application of Monte Carlo methods is justified from a certain complexity of the geometry onwards and has been discussed previously in the literature [8–11]. Monte Carlo methods can be used as purely numerical integration techniques for the calculation of the view factors. Since Monte Carlo integration involves considerable computations this type of applications can only be justified in a furnace simulation when the integrations do not appear inside an iteration loop.

Further, the number of integrations can be reduced by introducing maximum symmetry into the zoning of the geometrical system. Analogous geometrical configurations of zones are then obtained resulting in simple relations between certain view factors. Some of the view factors can then be calculated directly from the others, without necessitating further integration. In this context, three principles mentioned in Hottel and Sarofim's work 'Radiative Transfer' [4, p. 59] can be very useful: the principle of reciprocity, the conservation principle and the Yamauti principle [12].

Consider now the matrix **F** of all the view factors in the system. Let the basic non zero view factors from

which the other elements in $\mathbf{F}$ can be calculated, constitute the vector $\mathbf{F}'$. In practice, only linear dependencies are considered, so that the relations between the vector and the matrix may be written

$$\sum_k a(i,j,k)F'(k) = F(i,j) \tag{2}$$

whereas, from the conservation principle

$$\sum_j F(i,j) = 1. \tag{3}$$

When the view factor associated with a given emitter are obtained by Monte Carlo integration and they are subsequently used to calculate other view factors by means of (2), the conservation equation (3) will not necessarily hold any more, because of the statistical errors. Consequently, the view factors will be inadequate for the calculation of heat balances for the zones. Therefore, a regression technique was developed for adapting the Monte Carlo results, $\mathbf{F}'^*$, which are estimates of the real values $\mathbf{F}'$, in such a way that (3) is satisfied. The criterion chosen for this adaptation was such that the deviation with respect to $\mathbf{F}'^*$ was kept minimal. When the vector of adapted values is represented by $\hat{\mathbf{F}}'$ this criterion can be written

$$(\hat{\mathbf{F}}' - \mathbf{F}'^*)^T \mathbf{D}(\hat{\mathbf{F}}' - \mathbf{F}'^*) \text{ minimal} \tag{4}$$

where $\mathbf{D}$ is the diagonal matrix of the reciprocals of the variances on the Monte Carlo results $\mathbf{F}'^*$. These variances are obtained from

$$\frac{1}{D(i,i)} = \frac{F'^*_{(i)}[1 - F'^*_{(i)}]}{n-1}$$

in which n is the number of samples by which the Monte Carlo results are obtained [13].

The constraints (3) are accounted for in the following way. Combine (2) and (3) to express the constraints solely in terms of the independent view factors $\mathbf{F}'$

$$\mathbf{BF}' = \mathbf{1} \tag{5}$$

where $\mathbf{1}$ represents a vector with elements 1 and where the elements of the matrix $\mathbf{B}$ are

$$b(i,k) = \sum_j a(i,j,k),$$

$\mathbf{B}$ may contain linearly dependent rows. Therefore, define a matrix $\mathbf{B}'$ containing the linearly independent rows of $\mathbf{B}$ only. That leads to

$$\mathbf{B}'\mathbf{F}' = \mathbf{1}'.$$

The solution of this equation certainly satisfies (5). The number of rows in $\mathbf{B}'$ is smaller than the number of columns. Matrix $\mathbf{B}'$ can therefore be decomposed into two matrices $\mathbf{B}'_0$ and $\mathbf{B}'_1$, such that B'_0 is a non-singular square matrix. The vector $\mathbf{F}'$ has to be decomposed accordingly to yield

$$\mathbf{B}'_0\mathbf{F}'_0 + \mathbf{B}'_1\mathbf{F}'_1 = \mathbf{1}'$$

from which the solution of $\mathbf{F}'_0$ is obtained

$$\mathbf{F}'_0 = \mathbf{c} + \mathbf{C}\mathbf{F}'_1 \tag{6}$$

with

$$\mathbf{c} = \mathbf{B}'^{-1}_0 \mathbf{1}'$$

$$\mathbf{C} = -\mathbf{B}'^{-1}_0 \mathbf{B}'_1.$$

These equations have to be satisfied by the smoothed values $\hat{\mathbf{F}}'$ too.

A similar decomposition of $\mathbf{F}'^*$, $\hat{\mathbf{F}}'$ and $\mathbf{D}$ yields a new form for the objective function

$$(\hat{\mathbf{F}}'_0 - \mathbf{F}'^*_0)^T \mathbf{D}_0(\hat{\mathbf{F}}'_0 - \mathbf{F}'^*_0) + (\hat{\mathbf{F}}'_1 - \mathbf{F}'^*_1)^T \mathbf{D}_1(\hat{\mathbf{F}}'_1 - \mathbf{F}'^*_1) = \text{minimal}. \tag{7}$$

Substitution of (6) for the smoothed values $\hat{\mathbf{F}}'_1$ and $\hat{\mathbf{F}}'_0$ into (7), taking the derivative and equating it to zero yields the optimal value for $\hat{\mathbf{F}}'_1$

$$\hat{\mathbf{F}}'_1 = (\mathbf{C}^T\mathbf{D}_0\mathbf{C} + \mathbf{D}_1)^{-1}[\mathbf{D}_1\mathbf{F}'^*_1 - \mathbf{C}^T\mathbf{D}_0(\mathbf{F}'^*_0 - \mathbf{c})]. \tag{8}$$

Equation (8), together with (6), yields the smoothed view factors $\hat{\mathbf{F}}'_1$ and $\hat{\mathbf{F}}'_0$ satisfying the constraints (3) or (5). Notice that the procedure is easily extended to situations where not only diffuse surface reflections but also specular reflection is accounted for. In the calculation procedure of Sarofim and Hottel [3] using ρ_{s_j} as the specular reflection component of the reflectivity, (3) should be replaced by

$$\sum_j (1 - \rho_{s_j}) F(i,j) = 1$$

while the elements of $\mathbf{B}$ in (5) should be redefined by

$$b(i,j) = \sum_j (1 - \rho_{s_j}) a(i,j,k).$$

Example

Consider radiation inside a cube. Since there are 6 surface zones the view factor matrix has 36 elements. Only 2 of these are independent and non-zero, however, the view factor between 2 adjacent surfaces $F'(1)$ and that between 2 opposing sides $F'(2)$. By independent Monte Carlo integrations a value of 0.190 was obtained for $F'^*(1)$ and of 0.195 for $F'^*(2)$. Each surface has one opposing surface and 4 adjacent surfaces. Obviously, (3) does not hold for the uncorrected values

$$F'^*(2) + 4F'^*(1) = 0.955 \neq 1.$$

In this example (5) written for the smoothed values has the following form:

$$\begin{vmatrix} 4 & 1 \\ 4 & 1 \\ 4 & 1 \\ 4 & 1 \\ 4 & 1 \\ 4 & 1 \end{vmatrix} \begin{vmatrix} \hat{F}'(1) \\ \hat{F}'(2) \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}$$

In this case

$$\mathbf{B}' = |4 \quad 1|$$

so that, with

$$\hat{\mathbf{F}}'_0 = |\hat{F}'(2)|\,; \quad \hat{\mathbf{F}}'_1 = |\hat{F}'(1)|\,; \quad \mathbf{c} = |1|\,; \quad \mathbf{C} = |-4|.$$

Equation (6) becomes

$$\hat{\mathbf{F}}'_0 = 1 - 4\hat{\mathbf{F}}'_1$$

and (7)

$$\hat{\mathbf{F}}'_1 = [\mathbf{D}_1\mathbf{F}'^*_1 - 4\mathbf{D}_0(\mathbf{F}'^*_0 - 1)]/[16\mathbf{D}_0 + \mathbf{D}_1].$$

Assuming $\mathbf{D}_1 = \mathbf{D}_0$ then leads to

$$\hat{F}'(1) = 0.2006$$

$$\hat{F}'(2) = 0.1976$$

and

$$\hat{F}'(2) + 4\hat{F}'(1) = 1.$$

It may be concluded that the condition in (3) is met, while the change of $\mathbf{F}'^*$ is kept to a minimum. Further, the adapted values are closer to the exact solution, which can be found e.g. in [4, p. 50].

$$F'(2) = \frac{2}{\pi}\left(\ln\frac{2}{\sqrt{5}} + \sqrt{8}\tan^{-1}\frac{1}{\sqrt{2}}\right) - 1 = 0.199825$$

$$F'(1) = 0.25 \times (1 - F'(2)) = 0.20004375.$$

3. VIEW FACTORS AMONG SURFACE ZONES IN A REAL MEDIUM

The radiation intensity in a wavelength interval in real media decreases along a beam path because the gas absorbs energy. This is accounted for by introducing the transmittance, so that the view factor in a real medium is written

$$(F_{ij})_r = \frac{1}{A_i}\int_{A_i}\int_{A_j}\frac{\tau(S)\cos\beta_i\cos\beta_j}{\pi S^2}\,\mathrm{d}A_j\,\mathrm{d}A_i. \quad (9)$$

The transmittance depends upon the beam length, but also upon the temperature and gas composition along the beam. Edwards *et al.* [14], Tien and Lowder [15] and Lin and Greif [16] have developed fundamental models for the transmittance. For practical situations only Hottel's simple law has been used so far [1].

It would be unpractical to recalculate the multiple integrals in (9) in each iteration used in the solution of the set of (1), which is non-linear in the temperatures. To avoid this, the transmittance can be removed out of the multiple integrals of (9), in other words a mean value $\bar{\tau}_{ij}$ has to be defined such that

$$(F_{ij})_r = \bar{\tau}_{ij}F_{ij},$$

$\bar{\tau}_{ij}$ has to account for the beam length distribution but has to be taken at some average temperature and composition. The error introduced by this averaging depends on the size of the zones in which the total volume is divided or on the importance of the gradients. Mathematically

$$\bar{\tau}_{ij} = \int_{S_{\min}}^{S_{\max}}\tau(S)D.D.F.(S)\mathrm{d}S \quad (10)$$

where *D.D.F.* is the differential distribution of beam lengths, i.e. the probability that an arbitrarily chosen value of beam length lies between S and $S + \mathrm{d}S$.

A large number of Monte Carlo simulations for various configurations has led us to propose the following empirical expression for *D.D.F.*

$$D.D.F.(y) = N(y)P(y) \quad (11)$$

where

$$y = \frac{S - S_{\min}}{S_{\max} - S_{\min}}$$

is a dimensionless beam length and

$$N(y) = y^{1/2}(1 - y^{1/2})\frac{(-\ln y)^n}{n!} \quad (12)$$

$$P(y) = a + by + cy^2 + dy^3 + \ldots. \quad (13)$$

The form chosen for $N(y)$ was suggested from the overall characteristics of the simulated *D.D.F.* $N(y)$ exhibits a maximum in the interval $0 < y < 1$. Further, it can be shown by de l'Hôpital's rule that $N(y)$ is zero for $y = 0$. It contains only one adjustable parameter, n, however and this proved to be insufficient.

To provide more flexibility $N(y)$ was multiplied with the polynomial $P(y)$, which contains a number of adjustable parameters, a, b, c, d, ... For a given configuration these parameters and also n have to be determined by fitting (11) to the real distribution. An estimate for the latter can be obtained simultaneously with the view factors from the Monte Carlo simulations.

A practical way of fitting (11) to the 'real' distribution is to identify the moments. Those of the 'real' distribution are, of course, easily obtained along with the Monte Carlo simulation of F_{ij}. The moments of $D.D.F.(y)$ given by (11) are defined by

$$M_m = \int_0^1 y^m N(y)P(y)\,\mathrm{d}y. \quad (14)$$

After substitution of $N(y)$ and $P(y)$ (14) becomes

$$M_m = aW_{m+1} + bW_{m+2} + cW_{m+3} + dW_{m+4} + \ldots \quad (15)$$

with

$$W_i = (i + \tfrac{1}{2})^{-1-n} - (i + 1)^{-1-n} \quad (16)$$

and $i = m + 1, m + 2, m + 3, m + 4 \ldots$.

To start with, however, the parameter n of $N(y)$ is determined by approximating the mean beam length $\bar{y}$ of the real configuration by the mean of $N(y)$ only.

More precisely, among the $N(y, n)$ leading to a mean value $\bar{N}$ close to $\bar{y}$, that $N(y, n)$ is retained which has a value for $\bar{N}$ just below $\bar{y}$. Mathematically: n is taken as the smallest integer larger than zero satisfying

$$\bar{y} \geq (2.5)^{-1-n} - 3^{-1-n} = \bar{N} = W_2. \quad (17)$$

Notice that the mean of $N(y)$ is nothing but M_1, when $a = 1$ and $b = c = d = 0$.

The coefficients of the polynomial function $a, b, c, d, \ldots$ are subsequently determined from $M_0, M_1, M_2, \ldots$. Of course, $M_0 = 1$, since the distribution is normalized, while $M_1 = \bar{y}$. When only the moments of zero and first order M_0 and M_1 are used, an alternate way can be followed to determine the coefficients of the polynomial, after proceeding as above for the parameter n. In that case a third order polynomial having one single real root at $y = 1$ is selected. This constraint, and M_0 and N_1 enable the calculation of 3 out of the 4 coefficients. The resulting polynomial $P(y)$ has three fixed points: $[1,0]$, $[y_1, P(y_1)]$ and $[y_2, P(y_2)]$. To calculate y_1 and y_2 first the three conditions are written as

$$\begin{vmatrix} 1 & 1 & 1 \\ W_1 & W_2 & W_3 \\ W_2 & W_3 & W_4 \end{vmatrix} \begin{vmatrix} a \\ b \\ c \end{vmatrix} = \begin{vmatrix} 0 \\ 1 \\ \bar{y} \end{vmatrix} - d \begin{vmatrix} 1 \\ W_4 \\ W_5 \end{vmatrix}. \quad (18)$$

Clearly, a, b and c are linear functions of $\bar{y}$ and d.

An additional constraint is required to determine the fourth coefficient, d. This constraint was derived from an inspection of the third order polynomial through the three points determined so far. It is clear from (11)–(14) that $P(y)$ is positive at least in a range of values of y. A condition, sufficient to ensure $P(y)$ to be positive over the entire interval (0, 1), is that $P(y)$ is a minimum in the point corresponding to the smaller of the two ordinates $P(y_1)$ and $P(y_2)$. The coefficient d then also becomes a linear function of $\bar{y}$, the mean beam length. If $\bar{y}$ is situated in the interval (0.0489, 1) it follows from (17) that the adjustable parameter n in $N(y)$ has to be unity. The condition that $P(y)$ has to be positive in the open interval $0 < y < 1$ is equivalent with requiring that $P(y)$ has no roots in that interval i.e. that it does not change its sign. This condition yields a quadratic relation in $\bar{y}$, from which it is derived by means of elementary algebra that $P(y)$ is positive when the mean beam length $\bar{y}$ is comprised between 0.1048 and 0.4668.

When $\bar{y} < 0.0489$ the parameter n takes on values exceeding one and $P(y)$ becomes negative in the interval (0, 1). In that case the simplified approach is no longer applicable.

In real configurations the mean beam length is usually comprised between the above mentioned limits, however. The next two tests illustrate the possibilities and accuracy of the formula proposed for the beam length distribution. In both cases the simplified fitting approach is used.

Test 1

The beam length distribution was generated for two quite different configurations by means of Monte Carlo simulation. The interval between the minimum and the maximum beam length was divided into a number of increments. The interval to which each generated beam pertained was determined and this led to the frequency distribution *D.D.F.*(y). The mean beam length $\bar{y}$ used in the calculation of the *D.D.F.*(y) was also determined from the simulation.

The configurations used in this test are configurations I and II of Fig. 1, which obviously have to lead to quite different beam length distributions. Figures 2 and 3 show the excellent fit of the Monte Carlo simulated distribution that can be achieved by means of the proposed expression (11). The deviation is primarily located in the maximum of the sharp peak of Fig. 2, but the accuracy of the distribution obtained from the Monte Carlo simulations for discrete values of y is also lower in this area.

Test 2

In the second test, view factors calculated from (9), (10) and (11) are compared with the results of Hottel and Cohen [2] (Fig. 4) and with direct Monte Carlo results (8) in Fig. 5. To allow for a comparison with Hottel and Cohen the gas was assumed to absorb radiation according to the simple exponential law of Hottel. In Figures 4 and 5 the view factors are shown as functions of the product of the gas absorption coefficient and the length of the side of the squares, kX. The view factors calculated by Hottel and Cohen for configurations I, II, III and IV of Fig. 1 are compared in Fig. 4 with the results obtained from (10) which involves numerical integration of the products of the transmittance and the beam length distribution. Again the agreement is excellent. Further configurations which were tested are those labeled V, VI and VII in Fig. 1. Although those configurations are rather special, the results shown in Fig. 5 are also in excellent agreement with those obtained by means of the Monte Carlo simulations.

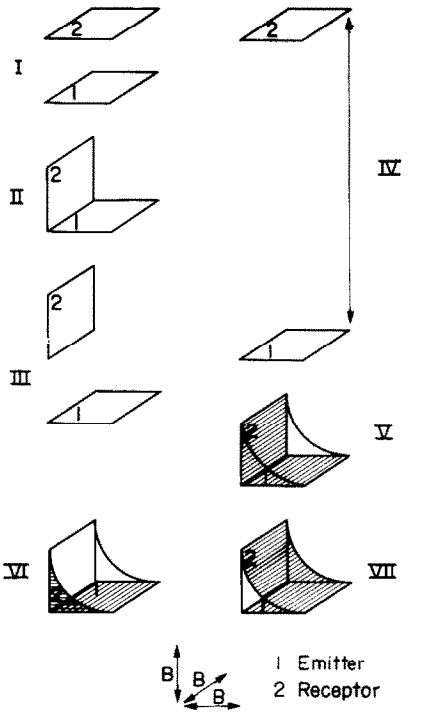

FIG. 1. Geometrical configurations for the test cases.

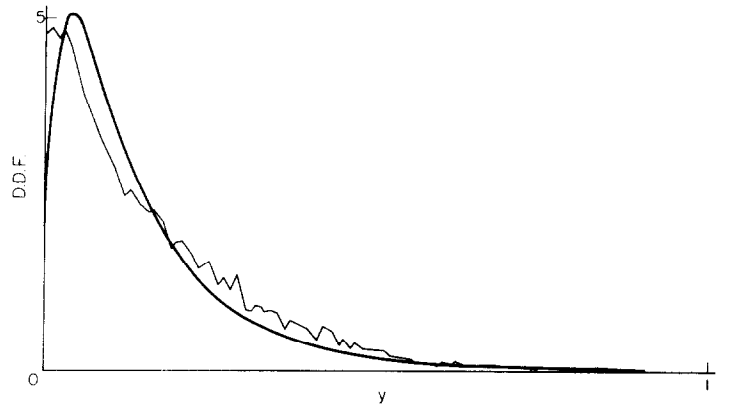

FIG. 2. Beam length distribution for configuration I. Broken line: simulated; smooth line: calculated.

4. DIRECT EXCHANGE AREAS

A third step in the calculation of the total exchange areas is the calculation of the direct exchange areas. Those between surfaces are obtained by simply multiplying the view factors by the corresponding emittor surfaces. To calculate the direct exchange areas between surface and volume zones, fictitious surfaces delimiting the gas volumes were introduced. The view factors towards those fictitious surfaces are also calculated in the real medium.

A constraint has to be set on the subdivision of the radiation space into zones: the radiation from a zone to a volume zone is allowed to intersect each fictitious surface of the volume zone in one direction only. In other words: none of the planes tangent to a fictitious surface is allowed to intersect another zone.

The direct interchange areas can now be calculated from

$$\overline{sg} = \mathscr{F} + \Sigma' \overline{ss_I} - \Sigma'' \overline{ss_I} - \Sigma \overline{ss_R} \qquad (19)$$

s_I represents the fictitious and s_R the real surfaces bounding the volume g. The term $\mathscr{F}$ is unity when the surface s is bounding g and zero when it is not. Σ' represents a summation over the surface, through which the radiation enters the volume g, Σ'' a summation over the surfaces through which the radiation leaves g.

For a given emitting zone (real, fictitious, surface or volume) it can be decided whether radiation enters or leaves the fictitious surfaces bounding the volume zones by drawing a straight line from a point on the emitter to a point on the fictitious surface and inspecting whether the line segment is entirely outside the volume or not.

The direct exchange areas between volumes are obtained from the same algorithm

$$\overline{g_1 g_2} = \Sigma' \overline{g_1 S_I} - \Sigma'' \overline{g_1 S_I} - \Sigma \overline{g_s s_R}.$$

When both volume and surface zones are gray the $\overline{gs}$ are obtained by means of the reciprocity principle

$$\overline{sg} = \overline{gs}$$

5. TOTAL EXCHANGE AREAS

To obtain the total exchange areas Hottel's procedure can be followed. The method is restricted to systems with gray gases and gray surfaces [1,6]. Reflected and transmitted radiation should have the same spectral wavelength distribution as the incident radiation. This constraint can be relaxed to a certain extent by considering the complete spectrum as consisting of a set of spectral bands with gray properties. The total exchange areas for the different bands can be summed after weighting them with respect to the

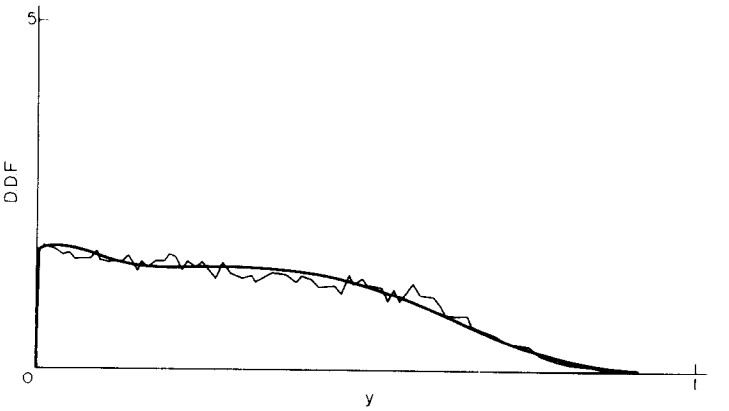

FIG. 3. Beam length distribution for configuration II. Broken line: simulated; smooth line: calculated.

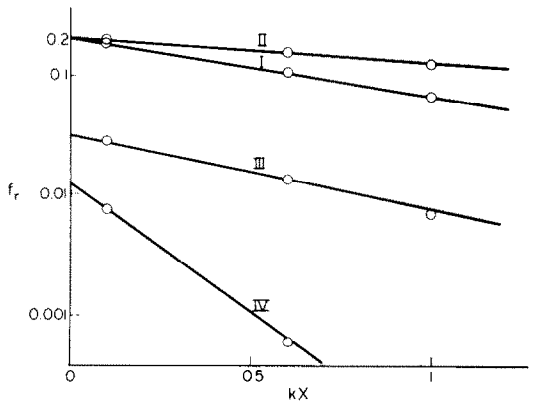

FIG. 4. View factors for configurations I, II, III and IV. Line: calculated; points: results of Hottel and Cohen [2].

relative importance of the radiation intensities in the bands. As shown already in the section on view factors in a real medium, a suitably chosen absorption model can be used in every spectral band.

6. OUTLINE OF CALCULATION OF TEMPERATURE DISTRIBUTION

The total sequence of calculations of the temperature distribution in a furnace by means of the zone method can be summarized as follows (Fig. 6).

After the real and fictitious surface zones have been defined the independent surface to surface view factors F' in a diatherm medium, the mean beam length and the higher order moments $M'_m(k)$ of the beam length distribution are determined by Monte Carlo simulation. The remaining surface to surface view factors are then calculated by the method outlined in Section 2. The corresponding mean beam lengths and the higher order moments $M_m(i, j)$ can also be calculated from the independent ones using the same relations (2), but after weighing with respect to the view factors

$$\sum_k a(i, j, k)F'(k)M'_m(k) = M_m(i, j).$$

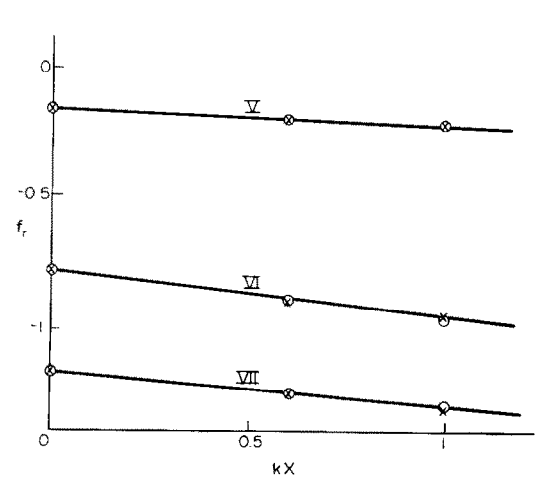

FIG. 5. View factors for configurations V, VI and VII. Line: model; points: Monte Carlo results.

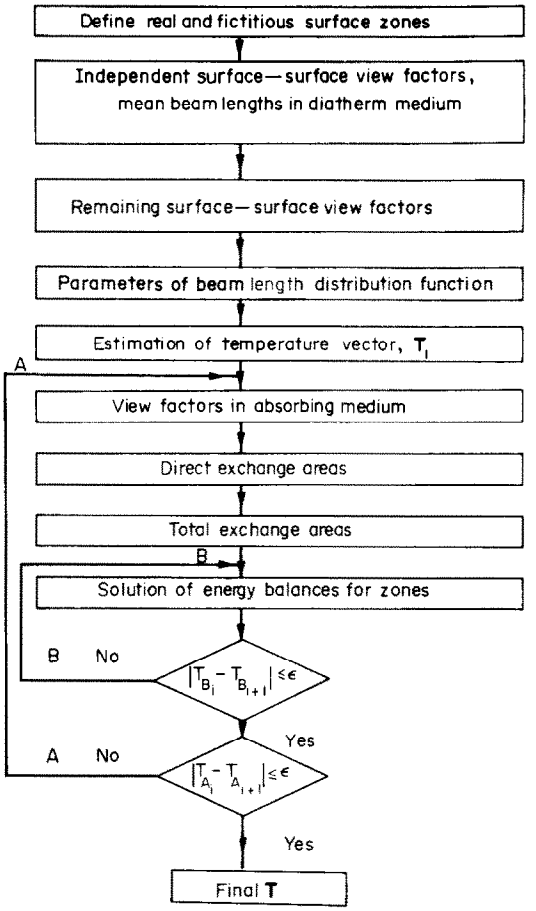

FIG. 6. Flow diagram for the calculation of the temperature distribution by means of the zone method.

The next step is the computation of the parameters of the *D.D.F.* of the beam length, for each surface to surface combination. A first estimate of the temperature vector in the furnace allows the calculation of the surface to surface view factors in the absorbing medium, using a suitable gas absorption model. The corresponding direct exchange areas are then easily obtained. Next, the remaining direct exchange areas (surface to volume, volume to surface, volume to volume) are determined. Then the set of energy balances (1) are solved for the temperatures and/or fluxes, by means of a Newton–Raphson routine e.g. if two subsequent iterations in the inner loop (A in Fig. 6) lead to identical temperature vectors, within the required degree of accuracy, the solution of the set of non-linear equations of (1) is considered to be achieved. The initial guess of the temperature vector used in the calculation of the surface to surface view factors in the absorbing medium has to be improved in a so-called outer iteration loop (B in Fig. 6). When two successive iterations in the B-loop agree the final temperature vector has been obtained.

7. CONCLUSIONS

The calculation procedures that are proposed improve the applicability of Monte Carlo methods to the

calculation of the total exchange areas entering into the zone method for furnace design. Inconsistencies resulting from statistical errors in the view factor matrix constructed from Monte Carlo results can be eliminated by a regression procedure. The introduction of an empirical equation for the beam length distribution keeps the multiple Monte Carlo integrations uncoupled from the temperature distributions in the radiation system and allows any absorption model for gases to be used. The beam length distributions can be calculated from the moments of the 'real' distribution function obtained by Monte Carlo simulation. An alternate and simple way for calculating the beam length distribution is shown to yield excellent results. The method has been applied in the simulation of a thermal cracking furnace [17].

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UNE METHODE VIABLE DE ZONAGE UTILISANT LES TECHNIQUES MONTE CARLO POUR LA SIMULATION DU RAYONNEMENT DANS LES FOURS INDUSTRIELS

Résumé — On propose des procédures de calcul dans lesquelles les méthodes de Monte Carlo peuvent être valablement utilisées pour calculer la matrice des facteurs géométriques entrant dans la méthode de Hottel. Des difficultés causées par des erreurs statistiques inhérentes peuvent être éliminées par régression. On propose une formule empirique pour la distribution de la longueur du rayon qui permet le calcul de l'échange des surfaces pour un modèle quelconque d'absorption du gaz (qui n'est pas limité à une loi d'absorption exponentielle) à partir des facteurs géométriques dans un milieu diathermane. Par cette procédure, les intégrations multiples, nécessaires pour le calcul des facteurs géométriques, sont découplées des distributions de température, de façon à les calculer une seule fois pour une configuration géométrique. L'utilité et l'applicabilité de la procédure sont illustrées par plusieurs exemples.

EINE VERBESSERTE ZONEN-METHODE BEI ANWENDUNG DES MONTE-CARLO-VERFAHRENS ZUR SIMULATION DER STRAHLUNG IN INDUSTRIEÖFEN

Zusammenfassung — Es werden Berechnungsverfahren vorgeschlagen, in denen die Monte-Carlo-Simulationsmethoden bequem angewandt werden können, um die Sichtfaktor-Matrix, die in der Zonenmethode von Hottel auftritt, zu berechnen. Schierigkeiten, die durch statistische Fehler verursacht werden, können mittels Regression elimineirt werden. Eine empirische Formel wird für die Strahllängenverteilung vorgeschlagen, mit der die gesamten Austauschflächen für jedes Gasabsorptionsmodell (d.h. das nicht auf ein exponentielles Absorptionsgesetz beschränkt ist) aus den Sichtfaktoren in einem strahlungsdurchlässigen Medium bestimmt werden könne. Durch dieses Verfahren werden die mehrfachen Integrationen, die für die Berechnung der Sichtfaktoren erforderlich sind, von den Temperaturverteilungen entkoppelt, so daß sie nur einmal für eine vorgegebene geometrische Konfiguration bestimmt werden müssen. Die Vorteile in der Anwendung dieser Methode werden durch zahlreiche Beispiele illustriert.

УСОВЕРШЕНСТВОВАННЫЙ ЗОНАЛЬНЫЙ МЕТОД С ИСПОЛЬЗОВАНИЕМ МЕТОДА МОНТЕ-КАРЛО ДЛЯ МОДЕЛИРОВАНИЯ ИЗЛУЧЕНИЯ В ПРОМЫШЛЕННЫХ ПЕЧАХ

Аннотация — Предложены способы расчёта, приспособленные для использования метода Монте-Карло для расчета матрицы коэффициентов облученности, входящей в зональный метод Хоттеля. Присущие методу статистические ошибки могут быть исключены регрессией. Предложена эмпирическая формула для расчета распределения длины пучка, с помощью которой можно определить суммарные площади обмена для любой модели поглощения в газе (не ограниченной законом экспоненциального поглощения) на основе коэффициентов облученности в диатермической среде. Используемый метод позволяет выполнять многократное интегрирование, необходимое для расчета коэффициентов облученности независимо от распределения температуры. Последнее, таким образом, можно рассчитывать лишь один раз для данной геометрии.

Продуктивность и границы применимости метода иллюстрируются рядом примеров.